CBCS SCHEME

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17CS36

Third Semester B.E. Degree Examination, Feb./Mar. 2022

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Write the converse, inverse and contrapositive for the following implication. For each, determine the truth value: "If 0 + 0 = 0 then 1 + 1 = 1" **(06 Marks)**

b. State the rule of universal specification. Show that the following argument is valid.

$$\begin{array}{l} \neg r(c) \\ \forall t([p(t) \to q(t)]) \\ \underline{\forall t([q(t) \to r(t)])} \\ \therefore \neg p(c) \end{array}$$

(06 Marks)

c. Prove the following without using Truth table:

i) $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$

ii) $p \to (q \to q) \Leftrightarrow (p \wedge q) \to r$ **(08 Marks)**

OR

2 a. If a proposition q has the truth value 1, determine all truth value assignment for the primitive propositions p, r and s for which the truth value of the following compound proposition is 1.

$[q \to \{(\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \to (\neg r \wedge q)\}$ **(06 Marks)**

b. Prove the statement "If n is an odd integer them n + 9 is an even integer" using :
(i) Direct method (ii) Indirect method (iii) Proof by contradiction **(06 Marks)**

c. Show that the following arguments are valid using Rules of inferences: **(08 Marks)**

i) $$\begin{array}{l} p \to q \\ r \to s \\ \underline{p \vee r} \\ \therefore q \vee s \end{array}$$

ii) $$\begin{array}{l} \neg p \leftrightarrow q \\ q \to r \\ \underline{\neg r} \\ \therefore p \end{array}$$

Module-2

3 a. Prove by mathematical induction

$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$ **(06 Marks)**

b. Find the number of permutations of the letters of the word MASSASAVGA. In how many of these all four A's are together? How many of them begin with S? **(06 Marks)**

c. Find the coefficient of :

(i) x^9y^3 in the expansion of $(2x-3y)^{12}$ (ii) x^0 in the expansion of $\left(3x^2 - \frac{1}{x}\right)^{15}$ **(08 Marks)**

OR

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. A box contains 15 IC chips of which 7 are defective and 8 are non-defective. In how many ways 5 chips can be chosen so that:
(i) All are non deflective (ii) All are defective (iii) 2 are non-defective **(06 Marks)**

c. In how many ways can 10 identical pencils are distributed among 5 children in the following cases:
(i) These are no restriction
(ii) Each child get atleast one pencil
(iii) The youngest child gets atleast 2 pencils. **(08 Marks)**

Module-3

5 a. Define relation. Let A and B be finite sets with |A| = 3. If these are 4096 relations from A to B, what is |B|? **(06 Marks)**

b. Define equivalence relation. Let S be the set of all non zero integers and $A = S \times S$ on A define the relation R by (a, b) R (c, d) iff ad = bc. Show that R is an equivalence relation. **(06 Marks)**

c. Define partial order relation. Let A = {1, 2, 3, 4, 6, 8, 12} and R be the partial ordering on A defined by aRb iff a divides b.
(i) Construct Hasse diagram
(ii) Find maximal and minimal elements
(iii) Find upper bounds and lower bounds if the subset B = {2, 3, 6} **(08 Marks)**

OR

6 a. Define one to one and onto functions. Let the set A = {a, b, c, d, e, f, g} and B= {1, 2, 3, 4}. How many onto functions exist from A to B? **(06 Marks)**

b. Let A = {1, 2, 3, 4} and let R be the relation on A defined by xRy iff "x divides y".
(i) Write the relation R (ii) Draw the digraph of R and matrix of R **(06 Marks)**

c. State pigeonhole principle. Prove that if 101 integers are selected from the set S = {1, 2, 3,….200} then atleast two of these are such that one divides the other. **(08 Marks)**

Module-4

7 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. Find the Rook polynomial for the 3×3 board using expansion formula. **(06 Marks)**

c. Solve the recurrence relation to find the n^{th} Fibonacci number. **(08 Marks)**

OR

8 a. Define Derangements. Find the number of derangements of 1, 2, 3, 4. **(06 Marks)**

b. How many integers between 1 and 300 (inclusive) are:
(i) Divisible by atleast one of 5, 6, 8? (ii) Divisible by none of 5, 6, 8? **(06 Marks)**

c. Four persons P_1, P_2, P_3 and P_4 who arrive late for a dinner party find that only one chair at each of 5 tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs. **(08 Marks)**

Module-5

9 a. Construct as optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(08 Marks)**

b. Write notes on: (i) Spanning subgraph (ii) Induced subgraph (iii) Isomorphic graphs **(12 Marks)**

OR

10 a. Define complete graph. Show that a complete graph with n vertices has $n(n-1)/2$ edges. **(06 Marks)**

b. What is a regular graph? If a graph with n vertices and m edges is K-regular. Show that $M = Kn/2$. **(06 Marks)**

c. Write notes on: (i) Euler circuits and Euler trails (ii) Connected and disconnected graph **(08 Marks)**